# DOWNLINK SCHEDULING IN CDMA DATA NETWORKS

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Abstract— We identify properties of optimal scheduling schemes for downlink traffic in a Code Division Multiple Access (CDMA) data-only network. Under idealised assumptions, we show that it is optimal for each base station to transmit to at most one delay-tolerant user at a time. Moreover we prove that a base station, when on, should transmit at maximum power for optimality. For a linear network, we characterise the optimal schedule as the solution to a linear program. As a by-product, our analysis yields bounds on throughput gains obtainable from downlink scheduling.

#### I. INTRODUCTION

Present mobile cellular systems allow only a limited capability for the transmission of data [3]. In fact, as far as CDMA is concerned, the IS-95A standard permits transmission only to a rate of approximately 10 kbits/s. This rate is too low to satisfy the delay requirements of many applications, including Web browsing and file transfers. Moreover, for current systems which support a large number of voice users, the network can be managed relatively simply on the basis of statistical averaging. This is no longer true when a few high data rate users are admitted into the system.

In view of the above difficulties, changes to existing CDMA standards have been proposed so as to permit high rate data traffic. One natural approach to resource management for such CDMA networks is to schedule the transmission of data user signals so as to avoid interference. The main idea behind this is that the increased transmission rate at lower interference more than compensates for the loss in available transmission time [4]. The simplest scheme in this direction is *intra-cell scheduling*, investigated in [1] and [5] for the uplink in a mixed voice and data system.

An alternative, more complex approach is to perform inter-cell scheduling, whereby base stations coordinate their communications with data users. A version of this approach has been examined and standardised in revision B of the IS-95 standard [2].

In this paper, we use analytic techniques to characterise optimal schedules for the downlink in CDMA networks. We focus on data-only networks and suppose that each user has a minimum rate requirement which must be satisfied. We assume a continuum of rates can be achieved, which are determined via the usual Gaussian interference model for the signal to interference ratio S/I [6]. Furthermore, we assume that the error rate requirement of a user is met provided his S/I is greater than some threshold. In our analysis we ignore any constraint on these rates placed by spreading, or by the bandwidth itself. However we do address this issue in our numerical experiments. We derive the following two optimality properties.

- 1. Each base station must transmit to its data users one at a time (for voice and data networks).
- 2. A base station must use full power when transmitting (for data networks only).

The paper is organised as follows. In Section II we introduce the basic model. Sections II-A and II-B identify optimal mobile scheduling and power allocation algorithms. The linear program that governs the optimal scheduling algorithm is introduced and analysed in Section III. Section IV examines a hybrid CDMA/Time-sharing scheme and our conclusions are presented in Section V.

# II. MODEL DESCRIPTION AND OPTIMALITY PRINCIPLES

We consider a network of cells, each cell containing one base station (BS). We adopt a standard model for the transmission of information from a BS to the users in its cell [6, Chapter 6]. Let W be the bandwidth and R the rate required by a user. If P is the power received by the user from the BS, and I the interference, then the user's energy per bit to noise density ratio  $E_b/N_0$ is

$$\frac{E_b}{N_0} = \frac{WP}{RI}.$$
 (1)

In order for the user to be able to decode the base station's transmission with an error probability  $\varepsilon$ , it is necessary that  $E_b/N_0 \ge \gamma$ , where  $\gamma = \gamma(\varepsilon)$  is some threshold determined by  $\varepsilon$ . We will use  $\gamma$  to denote the  $E_b/N_0$  requirement, assumed common to all users. We thus see that for a given received power P, the maximum rate  $\bar{R}$  achievable by a user is

$$\bar{R} = \frac{WP}{\gamma I}.$$
 (2)

Suppose there are  $N^k$  data users in the  $k^{\text{th}}$  cell  $E_k$ . Let  $P_T^k$  be the maximum available transmit power for the BS in  $E_k$ . Fix a time t, and suppose  $P^k$  is the total power at which BS  $E_k$  is transmitting at that time. Define  $\phi_i$  to be the fraction of power that is transmitted to the  $i^{\text{th}}$  data user in  $E_k$ , so that  $\sum_{i=1}^{N^k} \phi_i = 1$ .

For a particular cell  $E_k$ , let B(k) be the set of cells that are close enough to  $E_k$  that their base stations' transmissions cause interference to  $E_k$ . Typically one would expect B(k) to be the cells neighbouring  $E_k$ . Let  $\mathcal{G}_i^m$  be the path loss from the BS in  $E_m$  to user *i* in  $E_k$ . The interference caused to user *i* by its own BS' transmissions to other users in the cell is given by  $(1-\phi_i)P^k\mathcal{G}_i^k f_i$  where  $f_i$  is the orthogonality factor that represents the fraction of power transmitted to other users in the cell that appears as interference to user *i*.

If the BS uses orthogonal codes to transmit to distinct users, then intra-cell interference is virtually eliminated when the channel is Gaussian  $(f_i = 0)$ . However, when there is multi-path, this interference is only partially reduced and thus  $f_i \in (0, 1)$ . We denote the total external interference experienced by the  $i^{\text{th}}$  user due to transmissions of other BSs by  $I_i$ . Then

$$I_i \doteq \sum_{m \in B(k)} \mathcal{G}_i^m P^m$$

Using (2), we see that the rate data user i in cell k receives at time t is given by

$$\bar{R}_i = \frac{W}{\gamma} \times \frac{\phi_i P^k \mathcal{G}_i^k}{I_i + (1 - \phi_i) P^k \mathcal{G}_i^k f_i + \eta}, \qquad (3)$$

where  $\eta$  is the background noise.

The reader should note that the model assumes that the Rake receiver has negligible "self-noise" arising from multi-path.

## A. Mobile Scheduling

Consider the  $i^{\text{th}}$  data user in a particular cell  $E_k$  in the above cellular network model. For convenience, we drop the superscript k for the rest of this section. Suppose that under a pure CDMA strategy, a proportion  $a_i \in [0, 1]$  of the current total transmit power P is allocated to the  $i^{\text{th}}$  user. Then the rate which user i can obtain from CDMA is given by (3) with  $\phi_i$  replaced by  $a_i$ , so that

$$\bar{R}_{i}^{(CDMA)} = \frac{W}{\gamma} \times \frac{a_{i} P \mathcal{G}_{i}}{I_{i} + (1 - a_{i}) P \mathcal{G}_{i} f_{i} + \eta}.$$
 (4)

This is the throughput per unit time of user i using CDMA, since the rate is unchanging.

Now suppose that the BS transmits all the power used in the CDMA strategy to the  $i^{th}$  user, but only for a fraction  $a_i$  of the time. During the time that the BS transmits to user i, there is no transmission to other data users in the cell, so user i receives no interference from other users in the cell. Since the BS still uses the same total power P for the users at any instant, the users in neighbouring cells do not notice any change in their interference power. This modified strategy can be thought of as an *intra-cell scheduling* strategy. With this strategy the rate received by user i is given by (3) with  $\phi_i = 1$ , so that

$$\bar{R}_i^{(Intra)} = \frac{W}{\gamma} \times \frac{P\mathcal{G}_i}{I_i + \eta}.$$

Hence the total throughput received by the  $i^{\text{th}}$  data user in unit time is  $a_i \bar{R}_i^{(Intra)}$ . The ratio of the throughput received in the intra-cell scheduling strategy to that in the CDMA strategy is given by

$$1+\frac{(1-a_i)P\mathcal{G}_if_i}{I_i+\eta}.$$

Thus we have shown that for delay-tolerant users, transmitting to one user at a time yields an improvement over transmitting to users simultaneously, provided  $f_i > 0$ . If  $f_i = 0$  then the CDMA throughput would be equal to the throughput of intra-cell scheduling.

#### B. Power Allocation

We now investigate at what power the BS should transmit to a user in order to optimise the throughput per unit time. Suppose that a particular BS  $E_k$ transmits at some intermediate power  $P^k \in [0, P_T^k]$  for a period of time  $\tau$ . We suppose that the power transmitted by every BS to each of its users is constant over this period. By the mobile scheduling result we may suppose  $P^k$  is allocated to one user  $i_k$  so that  $\phi_{i_k} = 1$ . By (3) the average rate over the period  $\tau$  is

$$\overline{R}_{i_k} = \frac{W}{\gamma} \times \frac{P^k \mathcal{G}_{i_k}^k}{I_{i_k} + \eta},\tag{5}$$

where  $I_{i_k}$  is the external interference given by

$$I_{i_k} = \sum_{m \in B(k)} P^m \mathcal{G}_{i_k}^m.$$

Suppose now that BS  $E_k$  transmits to user  $i_k$  at full power for a proportion  $P^k/P_T^k$  of the time, and at 0

power otherwise. Then user  $i_k$  maintains the same average rate over that period, whereas all other users  $i_{k'}$  increase their average rates to  $R_{i_{k'}} \ge \overline{R}_{i_{k'}}$  by the convexity of (5) in each  $P^m$ ,  $m \in B(k)$ . Thus the sum of the users' rates increases. This clearly also holds for any objective function that is increasing in the rates. Hence the possibility of *any* base station in the network using intermediate power is ruled out.

### III. LINEAR DATA NETWORKS

#### A. Network Model

Consider a linear array of cells with an arbitrary distribution of users in each cell. We assume that each base station interferes with the transmissions of only its two neighbouring cells, so that  $B(k) = \{k - 1, k + 1\}$ . Our earlier results show that for optimality, one can assume that each base station has on and off periods, during which the total transmit power is  $P_T^k$  and 0 respectively. It follows that when the BS  $E_k$  is transmitting to user *i*, the rate received by user *i* is

$$\bar{R}_{i} = \frac{W}{\gamma} \times \frac{P_{T}^{k} \mathcal{G}_{i}^{k}}{\delta_{k-1} P_{T}^{k-1} \mathcal{G}_{i}^{k-1} + \delta_{k+1} P_{T}^{k+1} \mathcal{G}_{i}^{k+1} + \eta}, \quad (6)$$

where  $\delta_k$  is 1 if BS  $E_k$  is on and 0 otherwise. We assume that the scheduling interval, common to all base stations, is of unit length. We refer to the fraction  $\theta^k$  of the interval that a BS  $E_k$  is on as its duty cycle.

Let the *L*-time of a cell be the time during which only its left neighbouring station is on. Similarly define the cell's *R*-time, 2-time and 0-time as illustrated in Figure 1. Suppose the BS in  $E_0$  is transmitting to user *i*, so



Fig. 1. On/Off states of cell E<sub>0</sub>'s neighbouring stations during its L-, R-, 0-, and 2-times.

that  $P_i \doteq P_T^0 \mathcal{G}_i^0$  is the power received by that user from its own BS. Moreover, let  $P_{li}$  be the interference power received by the *i*<sup>th</sup> user in cell  $E_0$  from the base station of  $E_{-1}$  whenever  $E_{-1}$  is on, so that  $P_{li} = P_T^{(-1)} \mathcal{G}_i^{(-1)}$ . Define  $P_{ri} = P_T^1 \mathcal{G}_i^1$  similarly. Let the fractions of the total time that user *i* in cell  $E_0$  receives transmission from its own base station in L-time and R-time be  $\tau_{li}$  and  $\tau_{ri}$  respectively. Likewise define  $\tau_{2i}$  and  $\tau_{0i}$ . Finally let  $\tau_l^k \doteq \sum_{i \in E_k} \tau_{li}$  be the total L-time of cell  $E_k$ . Define  $\tau_r^k$ ,  $\tau_0^k$  and  $\tau_2^k$  analogously.

During 2-time, the power from both cells  $E_{-1}$  and  $E_1$  interfere with the transmission of the base station  $E_0$  to user *i*. Using (6) the normalised rate  $R_{2i}$  that user *i* receives during 2-time satisfies

$$R_{2i} = \frac{\gamma \bar{R}_{2i}}{W} = \frac{P_i}{P_{li} + P_{ri} + \eta} = \frac{P_i}{\eta (1 + \beta_{li} + \beta_{ri})},$$

with the normalization  $\beta_{li} \doteq P_{li}/\eta$ ,  $\beta_{ri} \doteq P_{ri}/\eta$ .

We obtain analogous expressions for the (normalised) rates that can be achieved by user i during R-time, L-time, and 0-time. The normalised throughput  $T_i$  per unit time obtained by user i is

$$\frac{\tau_{0i}P_{i}}{\eta} + \frac{\tau_{1i}P_{i}}{\eta(1+\beta_{li})} + \frac{\tau_{ri}P_{i}}{\eta(1+\beta_{ri})} + \frac{\tau_{2i}P_{i}}{\eta(1+\beta_{li}+\beta_{ri})}.$$
 (7)

The network-wide schedule of transmissions can be expressed in terms of the time allocations  $\tau = \{(\tau_{li}, \tau_{ri}, \tau_{0i}, \tau_{2i}), i \in E_k, k \in Z\}$  of every user, where the time allocations satisfy

$$\tau_i^k \ge 0 \quad \text{and} \quad \theta^k \doteq \tau_l^k + \tau_r^k + \tau_0^k + \tau_2^k \le 1.$$
 (8)

However, every allocation  $\tau$  that satisfies (8) does not necessarily generate a network-wide schedule. A feasible schedule must satisfy certain additional compatibility constraints which arise from the geometry of the schedule, [7].

#### **B.** Optimal Scheduling Problem

Consider the situation where each user in each cell has a minimum throughput requirement over a certain duration which we take to be the scheduling interval.

Now let  $S \subset Z$  be a finite cluster of cells in the linear network,  $\mathcal{F}_S$  be the feasible space of time allocations for users in this cluster and let  $A_i$  denote the minimum throughput requirement of user *i*. One objective is *maximise total throughput*: i.e. find a feasible time allocation  $\tau \in \mathcal{F}_S$  to maximise

$$\sum_{j \in S} \sum_{i \in E_j} T_i,$$

subject to

$$T_i \geq A_i, i \in \bigcup_{j \in S} E_j.$$

Thus the optimal scheduling problem is a linear program.

# C. Structure of the Optimal Schedule

Consider the optimal scheduling problem and focus on a particular cell  $E_0$  in the network and its two neighbours  $E_{-1}$  and  $E_1$ . Index the users in cell  $E_0$ by  $1, \ldots, M$ , starting with the left-most user and proceeding towards the right. Under conditions such as those below,  $\beta_{li}$  and  $\beta_{ri}$  satisfy the following monotonicity properties.

Assumption 1:

(a) If i > j then  $\beta_{ri} > \beta_{rj}$  and  $\beta_{li} < \beta_{lj}$ . (b) If user *i* is closer to the base station of cell  $E_0$  than user *j*, then  $\beta_{li} + \beta_{ri} < \beta_{lj} + \beta_{rj}$ .

The first property is true for any monotonic path loss function, and the second can be shown to hold for a power law path loss function.

Consider any optimal network-wide allocation  $\tau^*$  of transmission intervals to the various users, and let the transmission times allocated to user *i* in cell  $E_0$  be  $\tau_{ii}^*, \tau_{ri}^*, \tau_{0i}^*, \tau_{2i}^*$ . The following proposition provides some conditions that must be satisfied by an optimal solution.

Proposition 1: (a) If i > j, then  $\tau_{ri}^* > 0$  implies  $\tau_{lj}^* = \tau_{0j}^* = \tau_{2j}^* = 0$ , and  $\tau_{lj}^* > 0$  implies  $\tau_{ri}^* = \tau_{0i}^* = \tau_{2i}^* = 0$ . (b) If user j is closer to the base station of cell  $E_0$  than user i, then  $\tau_{2i}^* > 0$  implies  $\tau_{0j}^* = 0$ , and  $\tau_{0j}^* > 0$  implies  $\tau_{2i}^* = 0$ .

Part (a) implies that there is an outer set of users in the left part of cell  $E_0$  which receive data only when the right interferer  $E_1$  is on, and a similar outer set of users exists in the right part of cell  $E_0$ . Part (b) implies that if cell  $E_0$  needs to transmit when both interferers are on, then it would prefer to do so to users which are closer to the base station. The optimal solution can be characterised by specifying the leftmost user that has non-zero R-time, the rightmost user that has non-zero L-time, and the innermost users (possibly one on either side of the base station) that have non-zero 0-time, [7]. The proofs of these assertions are exchange arguments that rely on the monotonicity of the path gains stated in Assumption 1.

The above proposition is true even when the duty cycles and relative shifts between the transmission intervals of cells  $E_{-1}$ ,  $E_0$ , and  $E_1$  are all fixed and only the allocations to users within each cell can be adjusted. When the network is symmetric (that is the distribution of users in each cell is identical, but arbitrary), there exists a simple algorithm for obtaining the network-wide optimal schedule which determines the *inter-cell scheduling* bound [7].

#### IV. A HYBRID CDMA/TIME-SHARING SCHEME

We now confine ourselves to symmetrical networks, and present numerical results for a hybrid CDMA/Time-sharing scheme for various values of the orthogonality factor f. These results are compared with the analytic intra-cell scheduling bounds obtained by solving the optimisation problem described in Section III-B, under the assumption that the duty cycle of all base stations is 1. In the hybrid CDMA/Timesharing scheme each base station transmits to an inner set of users in CDMA mode for part of the time, and transmits to the rest of the users in time-sharing mode in the remaining time. By making the inner set large enough we can ensure the spreading rate constraint  $R_{max} = 0.1W$  is met. The time allocated to these inner users then depends on their minimum rate requirement. The orthogonality factor determines the interference between users that receive transmission simultaneously, and hence governs the minimum size of the CDMA set.

We assume there are 32 users uniformly spaced throughout the cell. The inner set consists of the N innermost users, and we will let  $\rho$  denote the fraction N/32. Note that  $\rho = 1$  corresponds to conventional CDMA and  $\rho = 0$  corresponds to intra-cell scheduling. We assume all users have a common minimum throughput requirement.

Figure 2 plots the maximum common minimum throughput achievable with this scheme against the fraction  $\rho$  of the total number of users that are in the CDMA set. Note that the leftmost point of each curve



Fig. 2. Plot of Maximum Common Throughput of Hybrid CDMA/Time-sharing scheme vs. Fraction of Total Number of Users that are in the CDMA set.

corresponds to the minimum size that the CDMA set must have for that value of the orthogonality factor, in order that the  $R_{max}$  constraint is met. It is seen that with the size of the CDMA set chosen to be this minimum value, this hybrid CDMA/Timesharing scheme approaches within 15% of the intra-cell scheduling bound.

# V. CONCLUSIONS

We established two optimality principles (i) each BS must transmit to at most one data user (holds in voice/data networks) and (ii) that each BS, when on, should transmit at full power. We then specialised to linear data networks and demonstrated how the optimal scheduling algorithm can be expressed as the solution to a linear program. Our numerical results showed that the bounds obtainable from this linear program lie close to achievable performance.

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